Comparing Graphs of Linear Motion

1.4

Cheetahs are adapted for speed—they are the fastest land animals. They can accelerate at faster rates than most sports cars (**Figure 1**). Cheetahs have been measured accelerating at rates greater than 10 m/s^2. To put this in perspective, a sports car can accelerate at approximately 7.2 m/s^2. In fact, cheetahs are capable of accelerating from rest to 10 m/s in only three strides.

You have already seen how position–time and velocity–time graphs can be used to analyze the linear motion of objects. In this section, we will introduce acceleration–time graphs and use all three types of graphs to analyze motion in more detail.

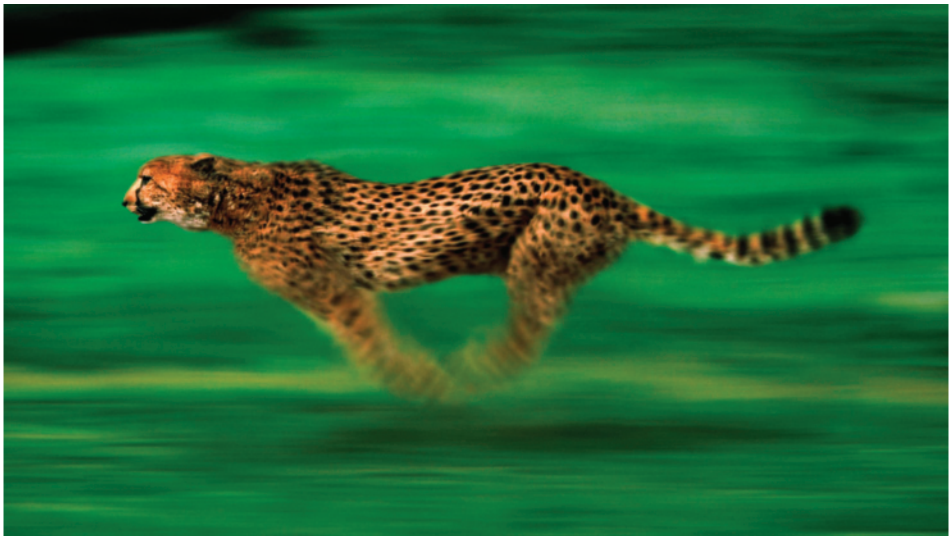

Figure 1 Cheetahs have the greatest acceleration of any animal.

Acceleration–Time Graphs

Earlier in this chapter, you learned how to find the displacement, or change in position, of an object by determining the area under a velocity–time graph. In a similar way, we can determine the change in velocity of an object from the area under an **acceleration–time graph**, which has acceleration on the vertical axis and time on the horizontal axis.

acceleration–time graph a graph describing motion of an object, with acceleration on the vertical axis and time on the horizontal axis

Consider the acceleration–time graph in **Figure 2**, which shows the motion of a cheetah. The points plotted on this graph lie along a horizontal straight line with a non-zero *y*-intercept. The acceleration is a constant 4.0 m/s^2, so this graph represents uniform acceleration.

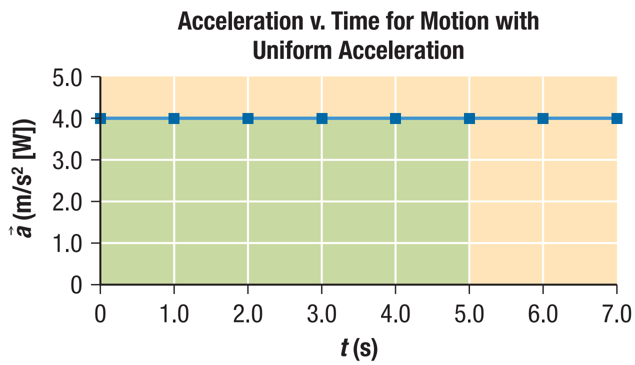

Figure 2 Acceleration–time graph showing motion with uniform acceleration

Investigation 1.4.1

Uniform Velocity (p. 47)
In this investigation, you will use a motion sensor to generate different types of motion graphs for an object moving with uniform velocity, and analyze these graphs.

If we calculate the area under the acceleration–time graph in Figure 2 from 0 s to 5.0 s, we will be determining the change in velocity of the object from $t = 0$ s to $t = 5.0$ s:

$$A = lw$$
$$= (5.0\ \cancel{s})\left(4.0\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\ [\text{W}]\right)$$
$$A = 20\ \text{m/s}\ [\text{W}]$$

Since the units are metres per second, the area we calculated represents a change in velocity.

The area under an acceleration–time graph represents the change in velocity of an object.

If the initial velocity of the cheetah is zero (the object is at rest), the final velocity is equal to the change in velocity, 20 m/s [W]. If the initial velocity is 5 m/s [W], however, then the graph tells us that the final velocity is

$$5\ \text{m/s [W]} + 20\ \text{m/s [W]} = 25\ \text{m/s [W]}$$

The graph does not tell us what the initial and final velocities are; it just tells us the change in velocity that occurs in the time interval.

Relationships among Linear Motion Graphs

Investigation 1.4.2

Motion Down a Ramp (p. 48)
In this investigation, you will use a motion sensor and different types of motion graphs to analyze the motion of an object rolling down a ramp.

Graphical analysis is one of the most powerful analytical tools available to physicists. In studying the motion of objects, analyzing position–time, velocity–time, and acceleration–time graphs can help us gain insight into real-life events such as the motion of the cheetah shown in Figure 1. This is particularly important because most objects in nature do not come equipped with a speedometer.

Figure 3 compares the three types of graphs of linear motion. All three graphs represent the same type of motion: uniform acceleration. Nevertheless, the three graphs look very different. When analyzing a motion graph, you may read information directly from the graph or determine further information by calculating the slope or the area of the graph.

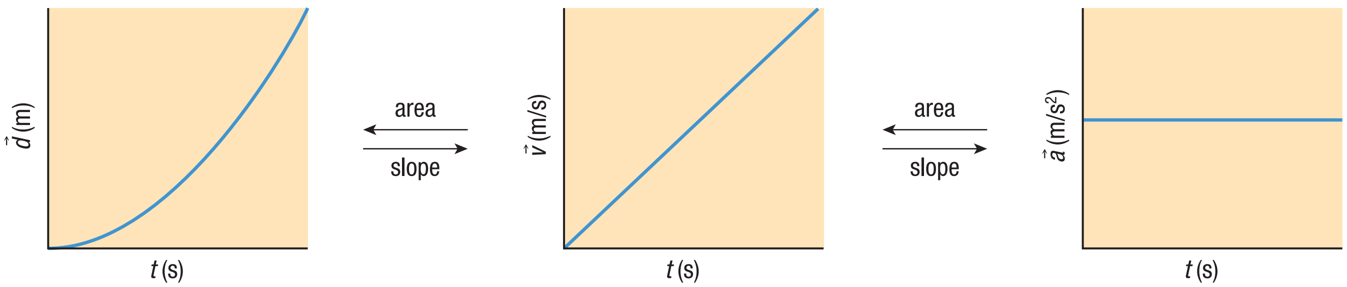

Figure 3 Position–time, velocity–time, and acceleration–time graphs of the same motion

Tutorial 1 Creating One Type of Motion Graph from Another

By using the information in **Figure 4**, we can analyze an acceleration–time graph further and get more information about the motion it describes.

Sample Problem 1: Creating a Velocity–Time Graph from an Acceleration–Time Graph

Use the acceleration–time graph in **Figure 4** to generate velocity and time data for the object. Then use these data to plot a velocity–time graph.

Step 1. To generate the velocity–time data, first calculate the area under the graph for several time points in Figure 4. Since the line is horizontal, we use the formula for the area of a rectangle, $A = lw$.

For Figure 4, $l = t$ (s); $w = \vec{a}$ (m/s^2 [W]); and $A = \vec{v}$ (m/s [W]), so in calculating $A = lw$, we are actually calculating $\vec{v} = (\Delta\vec{a})(\Delta t)$

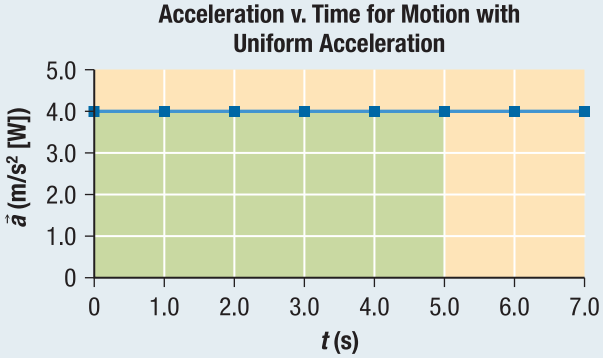

Figure 4 Using an acceleration–time graph to create other motion graphs

Step 2. **Table 1** shows the calculations for the area under the graph at 1 s intervals from $t = 0$ s to $t = 5.0$ s.

Table 1 Calculating the Velocity at Various Time Points in Figure 4

Time t (s)	Acceleration $\vec{a}$ (m/s^2 [W])	Equation $\vec{v} = (\Delta\vec{a})(\Delta t)$	Velocity $\vec{v}$ (m/s) [W]
0	4.0	$\vec{v} = \left(4.0\,\frac{\text{m}}{\text{s}^{\not{2}}}\,[\text{W}]\right)(0\,\not{\text{s}})$	0
1.0	4.0	$\vec{v} = \left(4.0\,\frac{\text{m}}{\text{s}^{\not{2}}}\,[\text{W}]\right)(1.0\,\not{\text{s}})$	4.0
2.0	4.0	$\vec{v} = \left(4.0\,\frac{\text{m}}{\text{s}^{\not{2}}}\,[\text{W}]\right)(2.0\,\not{\text{s}})$	8.0
3.0	4.0	$\vec{v} = \left(4.0\,\frac{\text{m}}{\text{s}^{\not{2}}}\,[\text{W}]\right)(3.0\,\not{\text{s}})$	12
4.0	4.0	$\vec{v} = \left(4.0\,\frac{\text{m}}{\text{s}^{\not{2}}}\,[\text{W}]\right)(4.0\,\not{\text{s}})$	16
5.0	4.0	$\vec{v} = \left(4.0\,\frac{\text{m}}{\text{s}^{\not{2}}}\,[\text{W}]\right)(5.0\,\not{\text{s}})$	20.0

Step 3. Plot the data to create a velocity–time graph (**Figure 5**).

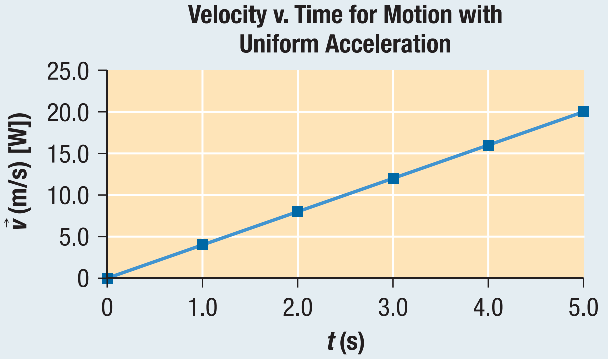

Figure 5 Velocity–time solution graph

Figure 5 shows the resulting graph. It is an increasing straight line with a zero intercept. It describes precisely the same motion that was described by the acceleration–time graph in Figure 4. Both graphs describe uniform acceleration.

Sample Problem 2: Creating an Acceleration–Time Graph from a Velocity–Time Graph

Use the velocity–time graph shown in **Figure 6** to plot the corresponding acceleration–time graph.

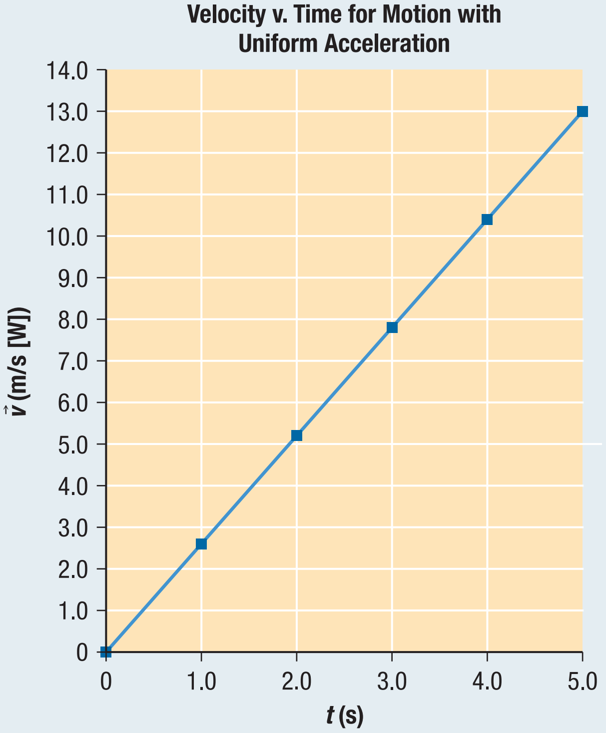

Figure 6 Given velocity–time graph

Step 1. The data plotted on the velocity–time graph in Figure 6 form an increasing straight-line graph with a zero intercept. You can determine acceleration from a velocity–time graph by calculating its slope. Since the velocity–time graph in Figure 6 is a straight line, its slope does not change. So we can calculate the slope or acceleration over any time interval.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

$$= \frac{7.8 \text{ m/s [W]}}{3.0 \text{ s}}$$

$$\vec{a}_{av} = 2.6 \text{ m/s}^2 \text{ [W]}$$

Step 2. Use this value to create an acceleration–time graph **(Figure 7)**.

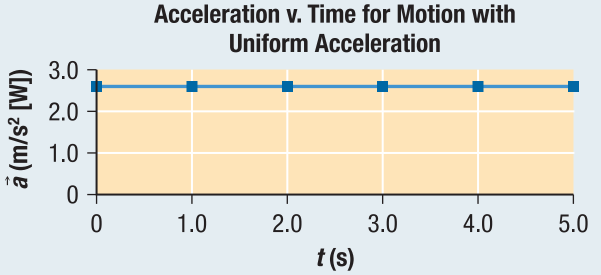

Figure 7 Acceleration–time solution graph

Figure 7 shows the corresponding acceleration–time graph. This graph shows a horizontal straight line with a *y*-intercept of 2.6 m/s^2 [W].

Note: If a velocity–time graph is not a straight line, you will need to determine the slope of the tangent for each time data point, and then use these data to plot the corresponding acceleration–time graph.

Practice

1. Generate position–time and acceleration–time data representing the motion of the object shown in **Figure 8**. Use the data to plot the corresponding position–time and acceleration–time graphs. T/I C

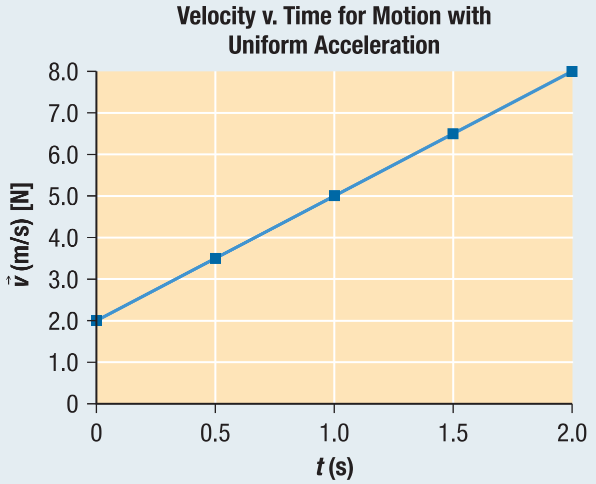

Figure 8

1.4 Summary

- The area under an acceleration–time graph gives the velocity of the object.
- Given one type of motion graph, you can read or calculate data from it in order to construct a different type of graph.

1.4 Questions

1. Copy and complete **Table 2** in your notebook by adding a check mark in each column that applies. K/U

Table 2

How do you determine ...	Given a ...	Read information from graph	Take the slope	Find the area
position	position–time graph			
velocity	position–time graph			
velocity	velocity–time graph			
velocity	acceleration–time graph			
acceleration	velocity–time graph			
acceleration	acceleration–time graph			

2. From the velocity–time graph in **Figure 9**, generate position–time data and then plot the corresponding position–time graph, assuming the initial position is 0 m. T/I C

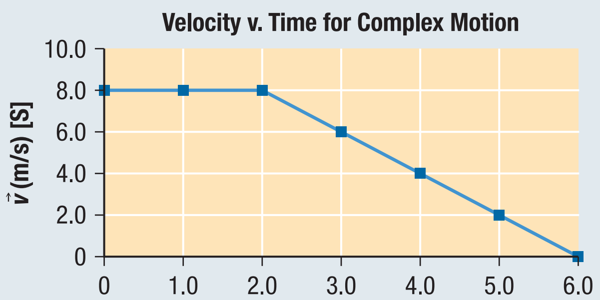

Figure 9

t (s)

3. Consider the position–time graph shown in **Figure 10**. T/I
   (a) What is the position of the object at $t = 5.0$ s?
   (b) What is the instantaneous velocity of the object at $t = 3.0$ s?
   (c) What is the average velocity for the object's motion from 0 s to 6.0 s?

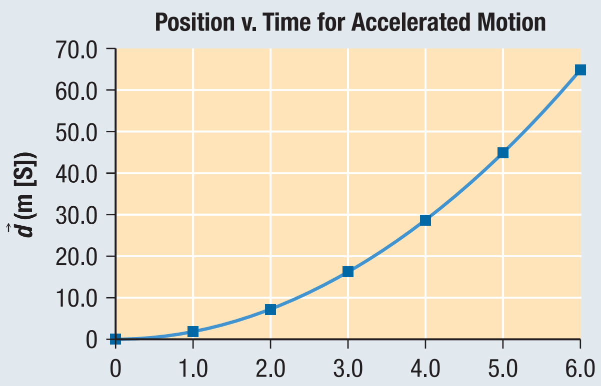

Figure 10

t (s)

4. Use the data in the velocity–time graph shown in **Figure 11** to plot the corresponding acceleration–time graph. T/I C

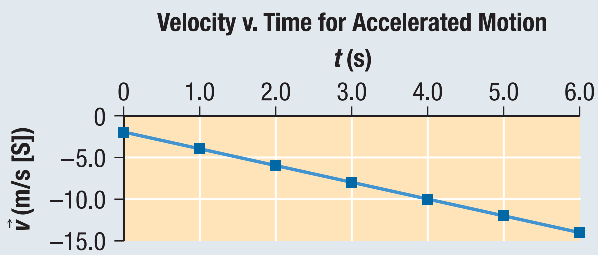

Figure 11