Chapter 7

Waves Transferring Energy

Practice Problems

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1. **Frame the Problem**
   - A metronome is undergoing periodic motion.
   - The frequency is the number of cycles per second.
   - The period is the time for one complete cycle.

   Identify the Goal
   There are two goals: frequency, f, and period, T.

   Variables and Constants

Involved in the Problem	Known	Unknown
number of beats, N	$N = 54$	f
time interval, Δt	$\Delta t = 55$ s	T
f		
T		

   Strategy
   Use definition equations for frequency, $f = \frac{N}{\Delta t}$ and period $T = \frac{\Delta t}{N}$.
   All needed variables are known, so substitute.
   $f = \frac{54}{55\text{ s}} = 0.98\text{ Hz}$
   $T = \frac{55\text{ s}}{54} = 1.02\text{ s}$
   The frequency is 0.98 Hz and the period is 1.02 s.

   Validate
   The metronome takes 55 s to beat 54 times, so one beat should take slightly longer than a second. It is beating at slightly less than one beat per second.

2. **Frame the Problem**
   - Butterflies wings are undergoing periodic motion.
   - Frequencies are given in beats per minute but are asked for in hertz.
   - Hertz is cycles per second, so minute must be converted to 60 s.

   Identify the Goal
   The range of frequency in hertz.

   Variables and Constants

Involved in the Problem	Known	Unknown
lowest frequency, f_l	$f_l = 450$ beats/min	f_l
highest frequency, f_h	$f_h = 650$ beats/min	f_h

   Strategy
   1 minute = 60 s
   Divide by seconds per minute
   $f_l = 450\text{ beats/min} = \frac{450\text{ beats}}{60\text{ s}} = 7.50\text{ Hz}$

$f_h = 650 \text{ beats/min} = \frac{650 \text{ beats}}{60 \text{ s}} = 10.8 \text{ Hz}$
The range of wing-beating frequencies for butterflies is from 7.50 Hz to 10.8 Hz.

Validate
600 times a minute, inside the given range, is about 10 times a second. The range of frequencies should be from a little below 10 times a second to a little above 10 times a second.

3. Frame the Problem
- A watch spring undergoes periodic motion, so has a frequency.
- The frequency is the number of cycles per second.

Identify the Goal
The time for 100 vibrations

Variables and Constants

Involved in the Problem	Known	Unknown
frequency, f	$f = 3.58$ Hz	Δt
number of vibrations, N	$N = 100$	
Δt		

Strategy
Use definition equations for frequency, $f = \frac{N}{\Delta t}$
Solve equation for Δt and substitute.
$\Delta t = \frac{N}{f} = \frac{100}{3.58 \text{ Hz}} = 29.7 \text{ s}$
The time for 100 vibrations is 29.7 s.

Validate
With a frequency of a little less than 4 oscillations per second, one oscillation should take a little more than $\frac{1}{4}$ s = 0.25 s. Therefore, it should take a little more than $\frac{1}{4} \times 100$ s, or 25 s, to make 100 oscillations.

4. Frame the Problem
- Swinging is a periodic motion.
- The frequency is the number of cycles per second.
- The period is the time for one complete cycle.

Identify the Goal
There are two goals: frequency, f, and period, T.

Variables and Constants

Involved in the Problem	Known	Unknown
number of swings, N	$N = 12$	f
time interval, Δt	$\Delta t = 30.0$ s	T
f		
T		

Strategy
Use definition equations for frequency, $f = \frac{N}{\Delta t}$ and period $T = \frac{\Delta t}{N}$.
All needed variables are known, so substitute.
$f = \frac{12}{30 \text{ s}} = 0.40 \text{ Hz}$
$T = \frac{30 \text{ s}}{12} = 2.5 \text{ s}$
The frequency of the swinging is 0.40 Hz and the period of swing is 2.5 s.

Validate
At 12 swings in 30 seconds, the child is taking over 2 s for each swing, and making less than half a swing per second.

Solutions for Practice Problems

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5. **Frame the Problem**
- A wave travelling in a spring has wavelength, frequency, and speed.
- The universal wave equation applies.
- Given a wave's speed, use $v = \frac{\Delta d}{\Delta t}$ to calculate the time to cover a distance.

Identify the Goal
The speed, v, of the wave and the time, Δt, to cover a given distance.

Variables and Constants

Involved in the Problem	Known	Unknown
length of spring, Δd	$\Delta d = 6.0$ m	v
frequency, f	$f = 10.0$ Hz	Δt
wavelength, λ	$\lambda = 0.75$ m	
speed, v		
time, Δt		

Strategy
Convert wavelength from cm to m.
Use universal wave equation to find the speed of the wave.
Use $v = \frac{\Delta d}{\Delta t}$ to find the time.

$$\begin{aligned} v &= f\lambda \\ &= 10.0 \text{ Hz} \times 0.75 \text{ m} \\ &= 7.5 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v &= \frac{\Delta d}{\Delta t} \\ \Delta t &= \frac{\Delta d}{v} \\ &= \frac{6.0 \text{ m}}{7.5 \text{ m/s}} \\ &= 0.67 \text{ s} \end{aligned}$$

The speed of the wave in the spring is 7.5 m/s and the time to travel down the spring is 0.67 s.

Validate
The wavelength is three quarters of a metre so, if waves were continuously being sent, about 8 waves should fit in the spring. At 10 waves per second, it should take less than a second for the waves to travel down the spring, and the waves should be travelling at greater than the length of the spring per second.

6. **Frame the Problem**
- Radio waves have wavelength, frequency, and speed.
- The frequency is related to wavelength and speed by the universal wave equation.

Identify the Goal
The frequency, f, of the waves.

Variables and Constants

Involved in the Problem	Known	Unknown
wavelength, λ	$\lambda = 21$ cm $= 0.21$ m	f
velocity, v	$v = 3.0 \times 10^8$ m/s	
frequency, f		

Strategy
Convert wavelength from cm to m.
Use universal wave equation.
Solve for f and substitute.

$$f = \frac{v}{\lambda}$$
$$= \frac{3.0 \times 10^8 \text{ m/s}}{0.21 \text{ m}}$$
$$= 1.4 \times 10^9 \text{ Hz}$$

The frequency of the radio waves is 1.4×10^9 Hz.

Validate
Radio waves with wavelength about a fifth of a metre would, if travelling 1 m/s, vibrate about 5 times a second. With a speed of 3×10^8 m/s, they should vibrate about 15×10^8 times a second, or about 1.5×10^9 Hz.

7. Frame the Problem
- Tsunamis are waves, so the universal wave equation applies to them.
- A distance and a time are given, so we can calculate speed.

Identify the Goal
The frequency, f, of the tsunami.

Variables and Constants

Involved in the Problem	Known	Unknown
wavelength, λ	$\lambda = 640$ km	v
distance, Δd	$\Delta d = 3250$ km	f
time, Δt	$\Delta t = 4.6$ h	
velocity, v		
frequency, f		

Strategy
km must be converted to m, so velocity can be in m/s, which is needed to get hertz.
First find velocity.
Then use universal wave equation.
Solve for f and substitute.

$$v = \frac{d}{t}$$
$$v = \frac{3250\,000 \text{ m}}{(4.6 \text{ h} \times 3600 \text{ s/h})} = 196 \text{ m/s}$$
$$f = \frac{v}{\lambda}$$
$$f = \frac{196 \frac{\text{m}}{\text{s}}}{640\,000 \text{ m}} = 3.1 \times 10^{-4} \text{ Hz}$$

The frequency of the tsunami is 3.1×10^{-4} Hz.

Validate
(a) The tsunami travelled a little over 3000 km in about $4\frac{1}{2}$ hours, so its speed should be about 700 km/h, or about 200 m/s.

(b) A huge wavelength wave should have a very long period, so a very short frequency.

8. Frame the Problem

- Universal wave equation should apply to earthquake waves.
- A speed and wavelength are given, so we can calculate frequency.
- A new speed is given. The frequency stays the same, so there must be a new wavelength.

Identify the Goal

(a) The frequency, *f*, of the earthquake wave.
(b) The new wavelength after the speed change, λ_2.

Variables and Constants

Involved in the Problem	Known	Unknown
original wavelength, λ_1	$\lambda_1 = 523$ km	f
speed, v	$v = 4.60$ km/s	λ_2
frequency, f		
wavelength, λ_2		

Strategy

Solve universal wave equation for frequency and substitute.
Use universal wave equation with same frequency, new speed.
Solve for λ_2 and substitute.
Distances can be left in km because both v and λ_1 use km.

(a) $f = \frac{v}{\lambda_1} = \frac{4.60 \text{ km/s}}{523 \text{ km}} = 8.80 \times 10^{-3} \text{ Hz}$

(b) $\lambda_2 = \frac{v}{f} = \frac{7.50 \text{ km/s}}{8.80 \times 10^{-3} \text{ Hz}} = 852 \text{ km}$

The frequency of the slower wave is 8.80×10^{-3} Hz and the wavelength of the faster wave is 852 km.

Validate

(a) Earthquake waves travel several kilometres per second but have wavelengths hundreds of kilometers long. Therefore they must have frequencies of about a hundredth of a km. (8.8×10^{-3} Hz is approximately 10^{-2} Hz.)

(b) The faster wave has a longer wavelength.

9. Frame the Problem

- Piano string has frequency, so is a vibrating object.
- Universal wave equation should apply to piano strings.
- Speed and frequency are given, so we can calculate wavelength.
- A note one octave higher has twice the frequency.

Identify the Goal

(a) The wavelength of middle C.
(b) The wavelength of C one octave above middle C.

Variables and Constants

Involved in the Problem	Known	Unknown
frequency, f	$f = 256$ Hz	λ_1
speed, v	$v = 343$ m/s	λ_2
wavelength of middle C, λ_1		
wavelength of note one octave higher, λ_2		

Strategy

Solve universal wave equation for wavelength and substitute.

(a) $\lambda_1 = \frac{v}{f} = \frac{343\ \frac{m}{s}}{256\ Hz} = 1.340\ m$

Wavelength of middle C is 1.34 m

(b) (by direct calculation)

$\lambda_2 = \frac{v}{f} = \frac{343\ \frac{m}{s}}{2 \times 256\ Hz} = \frac{343\ \frac{m}{s}}{512\ Hz} = 0.670\ m$

(by ratio : double frequency, so halve wavelength)

$\lambda_2 = \frac{\lambda_1}{2} = \frac{1.340\ m}{2} = 0.670\ m$

Wavelength of C above middle C is 0.670 m.

Validate

The two methods of calculating the value of C above middle C agree.

Problems for Understanding

21. The period of the pendulum is 4.0 s. Frequency is the reciprocal of the period, so the frequency will be 0.25 Hz.

22. If the period of a wave is doubled, the wavelength must double, because the velocity remains the same.

23. Frequency is 4 Hz. Velocity is 1.78 m/s

$$\lambda = \frac{v}{f} = \frac{1.78\ \frac{m}{s}}{4\ Hz} = 0.44\ m$$

24. $T = \frac{1}{f} = \frac{1}{60}\ Hz = 1.67 \times 10^{-2}\ s$

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{343\ \frac{m}{s}}{60\ Hz} = 5.72\ m$$

25. (a) $f = \frac{60}{42\ s} = 1.429\ Hz = 1.4\ Hz$

(b) $v = f\lambda = 1.429\ Hz \times 2.6\ cm = 3.7\ m/s$

26. Fundamental mode of vibration occurs when a half-wavelength standing wave is present. Thus $\frac{\lambda}{2} = 1.0\ m$, so $\lambda = 2.0\ m$.

$$v = f\lambda$$

$$f = \frac{v}{\lambda} = \frac{3.2\ \frac{m}{s}}{2.0\ m} = 1.6\ Hz$$

27. $v = \frac{3700\ \text{km}}{(5.2\ \text{h} \times 3600\ \frac{\text{s}}{\text{h}})}$
$= 0.20\ \frac{\text{m}}{\text{s}}$
$\lambda = \frac{v}{f}$
$= \frac{0.20\ \frac{\text{m}}{\text{s}}}{2.9 \times 10^{-4}\ \text{Hz}}$
$= 682\ \text{m}$
$= 6.8 \times 10^{2}\ \text{m}$

28. First calculate the wave speed:
$v = \frac{0.50\ \text{km}}{2.00\ \text{min}} \times \frac{1000\ \frac{\text{m}}{\text{km}}}{60\ \frac{\text{s}}{\text{min}}}$
$= \frac{500\ \text{m}}{120\ \text{s}}$
$= 4.17\ \frac{\text{m}}{\text{s}}$

(a) $f = \frac{v}{\lambda}$
$= \frac{4.17\ \frac{\text{m}}{\text{s}}}{3.5\ \text{m}}$
$= 1.2\ \text{Hz}$

(b) $T = \frac{1}{f}$
$= 0.84\ \text{s}$

29. (a) $T = \frac{120\ \text{s}}{117}$
$= 1.0256\ \text{s}$
$= 1.03\ \text{s}$

(b) $\%\ \text{error} = \frac{(1.0256\ \text{s} - 1.000\ \text{s})}{1.000\ \text{s}} \times 100\%$
$= 2.56\%$

(c) After 1 year, clock will be 2.56% of a year slow, which is
$2.56\% \times (365\ \text{days} \times 24\ \frac{\text{h}}{\text{day}} \times 3600\ \frac{\text{s}}{\text{h}}) = 2.56\% \times 3.16 \times 10^{8}\ \text{s}$
$= 8.07 \times 10^{5}\ \text{s} = 224\ \text{hours} = 9.34\ \text{days}$

(d) By shortening the string a little (square root of (1/1.0256) of the original length), the period of the pendulum can be shortened slightly.